

Current-current Fermi-liquid corrections to the superconducting fluctuations on conductivity and diamagnetism

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We analyze the behavior of the superconducting-fluctuations contribution to diamagnetism and conductivity in a model system having current-current interactions. We show that in proximity to a Mott-insulating phase one recovers an overall suppression of the fluctuating contribution to the conductivity with respect to diamagnetism, in close analogy with recent experiments on the underdoped phase of cuprate superconductors.

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It is generally believed that, due to their low superfluid densities and short correlation lengths, superconducting fluctuations (SCF) in underdoped cuprates should be relevant for transport and thermodynamic properties. Such SCF have been widely highlighted in the pseudogap region by several experimental measurements, ranging from diamagnetism¹⁻⁴ and Nernst effect^{3,5} to paraconductivity⁶⁻⁸. In particular, the survival of a large Nernst signal up to temperatures much larger than the superconducting transition temperature T_c in the underdoped region has been interpreted as the evidence for vortex-like phase fluctuations with a Kosterlitz-Thouless (KT) character due to the quasi-two-dimensional (2D) nature of the system^{1,3}. At the same time, several authors^{6,7} claimed that paraconductivity in underdoped cuprates simply follows the T dependence expected for the quasi-2D Aslamazov-Larkin (AL) regime of gaussian Ginzburg-Landau (GL) SCF close to T_c ⁹. This outcome motivated various investigations in the attempt to explain the experimental data on Nernst signal and diamagnetism within a GL-like framework¹⁰⁻¹².

Regardless of the KT or GL character of fluctuations, there are two outcomes of the experiments that are not expected for conventional superconductors: (i) the range of temperatures where the fluctuation conductivity is observed does not always match the one where a sizeable Nernst signal has been reported^{7,8}, and (ii) the SCF contribution to the conductivity is about two orders of magnitude *smaller* than the fluctuating diamagnetism in the same system, as recently pointed out by Bilbro *et al.* in Ref. 13. To be more precise, we recall that in 2D the contribution of SCF to the conductivity $\delta\sigma$ and diamagnetism $\delta\chi_d$ can be expressed both within the GL⁹ and KT¹⁴ theory in terms of the superconducting correlation length $\xi(T)$ as:

$$\delta\chi_d = -\frac{k_B T}{\Phi_0^2 d} \xi^2(T) \quad \delta\sigma = \frac{e^2}{16\hbar d} \frac{\xi^2(T)}{\xi_0^2} \quad (1)$$

Here Φ_0 is the flux quantum, ξ_0 the low temperature correlation length and d is the thickness of the effective 2D system (i.e. the interlayer spacing for layered systems

as cuprates). Both $\delta\sigma$ and $\delta\chi_d$ diverge as T approaches T_c due to the increase of the correlation length $\xi(T)$, with a T dependence that is power-law within the GL approach and exponential within the KT theory. From Eq.s (1) one could express the ratio $\delta\sigma/\delta\chi_d$ as

$$\frac{\delta\sigma}{|\delta\chi_d|} = \frac{\Phi_0^2 e^2}{16\hbar k_B T \xi_0^2} \frac{\xi_\sigma^2(T)}{\xi_{\chi_d}^2(T)} \quad (2)$$

where ξ_σ , ξ_{χ_d} are the correlations length extracted from paraconductivity and diamagnetism measurements, respectively. Since one would expect that the same length scale is involved in both cases, $\xi_\sigma^2/\xi_{\chi_d}^2 = 1$, right above T_c , $\delta\sigma/\delta\chi_d$ should be of order $\sim 10^5 (\Omega \text{ A/T})^{-1}$, while experimentally it turns out to be two orders of magnitude smaller than this^{1,13} (see also discussion below Eq. (36)). Let us notice that the above discussion holds regardless the nature of the SCF so that the ratio between $\delta\sigma$ and $\delta\chi_d$ depends only on the properties of the system away from T_c .

In this paper we show that the quantitative disagreement between the SCF contribution to diamagnetism and conductivity could be understood as a consequence of current-current interactions in a doped Mott insulator. The possible relevance of this kind of interactions to the physics of cuprates has been suggested within several contexts, ranging from the gauge-theory formulation for the $t-J$ model¹⁵ to the theoretical approaches emphasizing the role of microscopic currents^{16,17}. As a paradigmatic example we focus on the $t-J$ model within the slave-boson approach, where the Hartree-Fock (HF) correction of the quasiparticle dispersion leads to a difference between the quasiparticle current and velocity in the usual Landau Fermi-liquid (FL) language¹⁸. The effect of this dichotomy on the GL functional for SCF can be accounted for within a general field-theory for the t -J model^{15,19}, which includes fluctuations both in the particle-particle (p-p) and particle-hole (p-h) channel. By computing the AL contribution to diamagnetism and conductivity we find that the role of Landau FL corrections differs in the static or dynamic limit. As a consequence the SCF contribution to diamagnetism

and conductivity scales with a different prefactor, leading to the suppression of paraconductivity with respect to fluctuation diamagnetism when the Mott insulator is approached, in analogy with experiments in cuprates.

Let us start from the slave-boson version of the $t - J$ model,

$$H = -t\delta \sum_{\langle i,j \rangle \sigma} (c_{i\sigma}^\dagger c_{j\sigma} + H.c.) + J \sum_{\langle i,j \rangle} \mathbf{S}_i \mathbf{S}_j \quad (3)$$

where t is the electron hopping, J is the exchange interaction between electron spins, $c_{i\sigma}^\dagger (c_{i\sigma})$ is the fermionic creation (annihilation) operator and $\mathbf{S}_i = \Psi_i^\dagger \frac{\boldsymbol{\sigma}}{2} \Psi_i$ is the spin operator, with $\Psi_i = \begin{pmatrix} c_{i\uparrow} \\ c_{i\downarrow} \end{pmatrix}$. The sum is extended over all the $\langle i, j \rangle$ nearest-neighbor pairs on a square lattice, and we use units such that the lattice spacing and $\hbar = c = e = 1$. In Eq. (3) the decomposition of the electron operator in a fermionic spinon and bosonic holon part has been already carried out¹⁵, and only the fermionic degrees of freedom have been retained. Here we neglect the boson fluctuations, and we assume that slave bosons are always condensed, leading to the suppression factor $\delta = 2x/(1+x)$ of the hopping, scaling with the doping x . The interaction term of the Eq.(3) contains contributions both from the p-p and the p-h channel. We introduce the operators

$$\Phi_\alpha^c(\mathbf{q}) = \sum_{\mathbf{k}\sigma} \cos \mathbf{k}_\alpha c_{\mathbf{k}+\mathbf{q}/2,\sigma}^\dagger c_{\mathbf{k}-\mathbf{q}/2,\sigma}, \quad (4)$$

$$\Phi_\alpha^s(\mathbf{q}) = \sum_{\mathbf{k}\sigma} \sin \mathbf{k}_\alpha c_{\mathbf{k}+\mathbf{q}/2,\sigma}^\dagger c_{\mathbf{k}-\mathbf{q}/2,\sigma}, \quad (5)$$

$$\Phi^\Delta(\mathbf{q}) = \sum_{\mathbf{k}} \gamma_d(\mathbf{k}) c_{-\mathbf{k}+\mathbf{q}/2,\downarrow} c_{\mathbf{k}+\mathbf{q}/2,\uparrow} \quad (6)$$

where $\alpha = x, y$ and $\gamma_d(\mathbf{k}) = \cos \mathbf{k}_x - \cos \mathbf{k}_y$, so the interaction term reads

$$\sum_{\langle i,j \rangle} \mathbf{S}_i \mathbf{S}_j = -g \sum_{\mathbf{q}} \left(\frac{1}{2} \sum_{\alpha} \Phi_{\alpha\mathbf{q}}^c \Phi_{\alpha\mathbf{q}}^c + \Phi_{\alpha\mathbf{q}}^s \Phi_{\alpha\mathbf{q}}^s \right) + \Phi_{\mathbf{q}}^{\Delta*} \Phi_{\mathbf{q}}^{\Delta} \quad (7)$$

with $g = 3J/4$. The RHS of Eq. (7) represents the interaction in the p-h density (Φ^c), p-h current (Φ^s) and p-p (Φ^Δ) channel, respectively²⁰.

We decouple (7) by means of the Hubbard-Stratonovich (HS) transformation both in the p-p and in the p-h channel. After the integration of the fermions the action reads:

$$S = \sum_{\mathbf{q}} \left(\sum_{\alpha} \frac{|\phi_{\alpha\mathbf{q}}^c|^2}{2g} + \frac{|\phi_{\alpha\mathbf{q}}^s|^2}{2g} \right) + \frac{|\Delta_{\mathbf{q}}|^2}{g} - \text{Tr} \log \hat{A}_{kk'}. \quad (8)$$

Here ϕ^c , ϕ^s and Δ are the HS fields, the trace acts over momenta, frequencies and spins, $k \equiv (\mathbf{k}, i\varepsilon_n)$, $q \equiv (\mathbf{q}, i\omega_m)$, and ε_n, ω_m are the Matsubara fermion and boson frequencies, respectively. The $\hat{A}_{kk'}$ matrix is defined

(in the usual Nambu notation) as

$$\begin{aligned} \hat{A}_{kk'} = & - \left[i\omega_n + \sin\left(\frac{\mathbf{k}+\mathbf{k}'}{2}\right)_\alpha \phi_{\alpha\mathbf{k}-\mathbf{k}'}^s \right] \hat{\tau}_0 \\ & + \left[\xi_{\mathbf{k}}^0 - \cos\left(\frac{\mathbf{k}+\mathbf{k}'}{2}\right)_\alpha \phi_{\alpha\mathbf{k}-\mathbf{k}'}^c \right] \hat{\tau}_3 - \left[\Delta_{\mathbf{k}-\mathbf{k}'} \gamma_d\left(\frac{\mathbf{k}+\mathbf{k}'}{2}\right) \right] \hat{\mathbf{q}} \end{aligned}$$

where $\xi_{\mathbf{k}}^0$ is the bare dispersion

$$\xi_{\mathbf{k}}^0 = -2t\delta(\cos \mathbf{k}_x + \cos \mathbf{k}_y) - \mu \quad (10)$$

The Eq. (9) can be decomposed as $\hat{A}_{k,k'} = -\hat{G}_0^{-1} \delta_{k,k'} + \hat{\Sigma}_{k-k'}$. Here $\hat{G}_0^{-1} \delta_{k,k'}$ contains the $q = 0$ saddle-point values ($\phi_0^c, \phi_0^s, \Delta_0$) of the HS fields obtained by minimization of the mean-field action

$$S_{MF} = \sum_{\mathbf{q}} \left(\sum_{\alpha} \frac{|\phi_{\alpha\mathbf{q}}^c|^2}{2g} + \frac{|\phi_{\alpha\mathbf{q}}^s|^2}{2g} \right) + \frac{|\Delta_{\mathbf{q}}|^2}{g} - \text{Tr} \log(\hat{G}_0^{-1}), \quad (11)$$

while $\hat{\Sigma}_{k-k'}$ contains the fluctuating parts of the HS fields. The standard GL functional above T_c will then be given by the expansion of Eq. (8) around the mean-field action (11) as $S_{GL} = \sum_n \text{Tr}[\hat{G}_0 \hat{\Sigma}]^n / n$, with $\Delta_0 = 0$. As far as the saddle-point values in the p-h channels are concerned, one can easily see that $\phi_0^s = 0$, while $\phi_0^c \neq 0$ satisfies the following self-consistent equation:

$$\phi_0^c = \frac{2g}{N} \sum_{\mathbf{k}} \cos \mathbf{k}_\alpha f(\beta \xi_{\mathbf{k}}) \quad (12)$$

where $f(x)$ is the Fermi function, $\beta = 1/T$ and $\xi_{\mathbf{k}}$ is the quasiparticle dispersion, given by the $\hat{\tau}_3$ term of Eq. (9):

$$\xi_{\mathbf{k}} = -(2t\delta + \phi_0^c)(\cos \mathbf{k}_x + \cos \mathbf{k}_y) - \mu \quad (13)$$

As one can see, the ϕ_0^c value corresponds thus to the standard HF correction of the quasiparticle dispersion.

In order to compute the contribution $\delta\chi(q)$ of SCF to the electromagnetic response function we need to introduce in the effective action also the electromagnetic potential \mathbf{A} . For the model (3) this can be done via the Pierls substitution $c_i^\dagger c_{i+\alpha} \rightarrow c_i^\dagger c_{i+\alpha} e^{-iA_i^\alpha}$, which modifies the $\hat{A}_{kk'}$ matrix with two additional contributions

$$- 2t\delta \sin\left(\frac{\mathbf{k}+\mathbf{k}'}{2}\right)_\alpha A_{k'-k}^\alpha \hat{\tau}_0 \quad (14)$$

$$+ t\delta \cos\left(\frac{\mathbf{k}+\mathbf{k}'}{2}\right)_\alpha A_{k'-k+s}^\alpha A_{-s}^\alpha \hat{\tau}_3. \quad (15)$$

Eq. (14) corresponds to the usual term $-\mathbf{J} \cdot \mathbf{A}$, where

$$J_\alpha \equiv \partial_{\mathbf{k}_\alpha} \xi_{\mathbf{k}}^0 = 2t\delta \sin \mathbf{k}_\alpha \quad (16)$$

is the quasiparticle current. Notice that in the presence of HF corrections to the quasiparticle dispersion (13) the quasiparticle current J_α is different from the quasiparticle velocity

$$v_\alpha \equiv \partial_{\mathbf{k}_\alpha} \xi_{\mathbf{k}} = (2t\delta + \phi_0^c) \sin \mathbf{k}_\alpha \quad (17)$$

In the usual Landau FL language, these two quantities are related by the Landau FL F_1^s corrections as

$J_\alpha = v_\alpha(1 + F_1^s/3)$ for an isotropic system in three dimensions¹⁸. As we shall see, in our approach the analogous role of Landau FL corrections will be played by the HS fields related to p-h fluctuations. Neglecting the diamagnetic contributions, that are not relevant for the following discussion, the leading terms in \mathbf{A} of the action $S_{GL}(\mathbf{A}, \Upsilon^{HS})$ ($\Upsilon^{HS} = \phi^c, \phi^s, \Delta$) can be written in a compact form as:

$$S_{GL}(\mathbf{A}, \Upsilon^{HS}) = -\frac{1}{2}\chi_{jj}^{\alpha\beta} A_q^\alpha A_{-q}^\beta + A_q^\alpha F^\alpha(\Upsilon^{HS}) \quad (18)$$

where $\chi_{jj}^{\alpha\beta}$ is the mean-field current-current correlation function

$$\chi_{jj}^{\alpha\beta}(q) = -\frac{2}{N} \sum_{\mathbf{k}} (\partial_{\mathbf{k}_\alpha} \xi_0)(\partial_{\mathbf{k}_\beta} \xi_0) \frac{f(\beta \xi_{\mathbf{k}+\frac{\mathbf{q}}{2}}) - f(\beta \xi_{\mathbf{k}-\frac{\mathbf{q}}{2}})}{i\omega_n + \xi_{\mathbf{k}+\frac{\mathbf{q}}{2}} - \xi_{\mathbf{k}-\frac{\mathbf{q}}{2}}} \quad (19)$$

and $F^\alpha(\Upsilon^{HS})$ is a function of the Υ^{HS} fields which describes the connection of the electromagnetic potential to the HS fields. The current-current response function $\Lambda^{\alpha\beta}(q)$ can be computed from the partition function $Z[\mathbf{A}] = \int \mathcal{D}\Upsilon^{HS} e^{-S_{GL}(\mathbf{A}, \Upsilon^{HS})}$ of the model as

$$\Lambda^{\alpha\beta}(q) = \left. \frac{\delta \ln Z[\mathbf{A}]}{\delta A_q^\alpha \delta A_{-q}^\beta} \right|_{A_q^\alpha = A_{-q}^\beta = 0} = \chi_{jj}^{\alpha\beta}(q) + \delta\chi_{jj}^{\alpha\beta}(q) \quad (20)$$

where $\chi_{jj}^{\alpha\beta}(q)$ simply follows from the quadratic term of Eq. (18), while $\delta\chi_{jj}^{\alpha\beta}(q)$ is the contribution coming from the fluctuating modes coupled to \mathbf{A} , and depends on the

explicit form of the $F(\Upsilon^{HS})$ function:

$$\delta\chi_{jj}^{\alpha\beta}(q) = \langle F^\alpha(\Upsilon^{HS}) F^\beta(\Upsilon^{HS}) \rangle \quad (21)$$

Starting from the current-current response $\Lambda^{\alpha\beta}(q)$ (20) the paraconductivity simply follows from the dynamic limit ($\mathbf{q} = 0, \omega \rightarrow 0$) after analytical continuation of the Matsubara frequency ω_m to the real frequency ω

$$\delta\sigma = [\text{Im} \delta\chi_{jj}^{\alpha\alpha}(\omega, \mathbf{q} = 0)/\omega]_{\omega \rightarrow 0}, \quad (22)$$

while the fluctuations contribution to the diamagnetism is connected instead to the static limit ($\mathbf{q} \rightarrow 0, \omega = 0$),

$$\delta\chi_d = -[\delta\chi_{jj}^t(\mathbf{q}, \omega = 0)/\mathbf{q}^2]_{\mathbf{q} \rightarrow 0}. \quad (23)$$

where $\delta\chi_{jj}^t$ is the transverse part of the fluctuation correction to the current-current response function. In the absence of fluctuations in the p-h channel, only the pairing field is coupled to \mathbf{A} so that $\mathbf{F}(\Delta) \sim \mathbf{p}\Delta_p^2$ and one recovers the standard AL correction (see Eq. (27) below). In our case, one immediately sees from Eq. (9) and (14) that the ϕ_α^s field appears in the actions with the same structure of the electromagnetic potential A^α , i.e. it is coupled to the fermionic current. Thus, we expect that it will contribute to the $\delta\chi_{jj}^{\alpha\beta}(q)$ correction (20) above.

With lengthy but straightforward calculations one obtains that the effective action (18) at leading order in the gauge and HS fields is given by

$$\begin{aligned} S_{GL}(\mathbf{A}, \phi^s, \Delta) = & -\frac{1}{2}\chi_{jj}^{\alpha\beta} A_q^\alpha A_{-q}^\beta - \frac{1}{2} \frac{\chi_{jj}^{\alpha\beta}}{2t\delta} (A_q^\alpha \phi_{-q}^{s\beta} + \phi_q^{s\alpha} A_{-q}^\beta) + \frac{1}{2} \left[g^{-1} - \frac{\chi_{jj}^{\alpha\beta}}{(2t\delta)^2} \right] \phi_q^{s\alpha} \phi_{-q}^{s\beta} + \\ & + \left[g^{-1} - \Pi(\omega) + c\mathbf{q}^2 \right] \Delta_q^* \Delta_q - c' \mathbf{q} \left(\mathbf{A}_{q'} + \frac{\phi_{q'}^s}{2t\delta} \right) (\Delta_q^* \Delta_{q-q'} + \Delta_{q+q'}^* \Delta_q) \end{aligned} \quad (24)$$

where summation over repeated indexes is implicit and only the terms relevant for the following discussion are included. The c and c' terms in Eq. (24) follow from the expansion to leading order in \mathbf{q} of the fermionic bubbles associated to the Δ^2 term and to the $\mathbf{A}\Delta^2$ and $\phi^s\Delta^2$ terms, respectively (see Fig. 1). In the former case for example one expands

$$\Pi(q) = -\frac{T}{N} \sum_{\mathbf{k}} \gamma_d^2(\mathbf{k}) \mathcal{G}_{\mathbf{k}+q/2} \mathcal{G}_{-\mathbf{k}+q/2} \simeq \Pi(\omega) - c\mathbf{q}^2. \quad (25)$$

where the quasiparticle Green's function, $\mathcal{G}_{\mathbf{k}} = (i\omega_n - \xi_{\mathbf{k}})^{-1}$, contains the full dispersion $\xi_{\mathbf{k}}$. As a consequence, c is proportional to the second-order derivative of \mathcal{G} , which in turn scales as $(\partial \xi_{\mathbf{k}_\alpha})^2 \equiv v_\alpha^2$. In the case of

the $c'\mathbf{q}$ term instead one carries out a single derivative of the fermionic bubble which contains already a current insertion \mathbf{J} , associated to each \mathbf{A} or ϕ^s field, see Eqs (9) and (14) and Fig. 1. Thus, we have in a short-hand notation:

$$c \propto (\partial \xi_{\mathbf{k}_\alpha})^2 \sim v_F^2, \quad c' \propto \partial \xi_{\mathbf{k}_\alpha} \partial \xi_{\mathbf{k}_\alpha}^0 \sim v_F J_F \quad (26)$$

where v_F, J_F are the average values on the Fermi surface. If we do not consider the interactions in the p-h channel leading to the HF correction of the band dispersion we would get $\xi_{\mathbf{k}} = \xi_{\mathbf{k}}^0$ (see Eq. (13)) and the velocity coincides with the current, so that $c = c'$. In this case, as mentioned above, the electromagnetic potential \mathbf{A} is coupled only with the pairing field Δ via

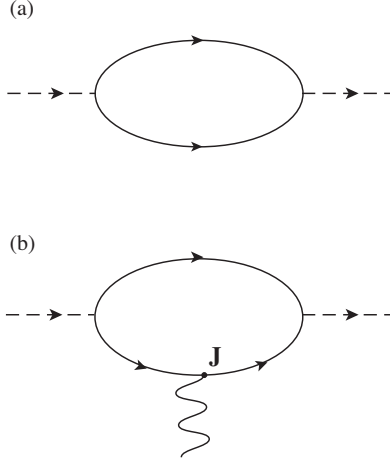


FIG. 1: Fermionic bubbles: (a) is the $\Pi(q)$ bubble associated to the Δ^2 term, while (b) is the fermionic bubble relative both to $\mathbf{A}\Delta^2$ and $\phi^s\Delta^2$ terms. The solid lines are the Green's functions \mathcal{G} containing the full quasiparticle dispersion ξ . The dashed lines are the pairing field Δ . The wavy line represents either the electromagnetic potential \mathbf{A} or the current field ϕ^s , both associated to the quasiparticle current $\mathbf{J} \propto \partial\xi^0$.

a term $\sim c\mathbf{A}\mathbf{p}\Delta_p^2$, so that the fluctuation correction to the current-current response function (20) is given by the usual AL contribution⁹ (see Fig.2):

$$\delta\chi^{\alpha\beta}(q) = c^2 T \sum_p (2\mathbf{p} + \mathbf{q})_\alpha (2\mathbf{p} + \mathbf{q})_\beta L(p) L(p+q) \quad (27)$$

where $L(p) = \langle \Delta_p^* \Delta_p \rangle$ is the propagator of the SCF. Since $c \propto v_F^2$ is constant, the SCF contribution to the diamagnetism (23) and conductivity (22) is in both cases proportional to v_F^4 , and the ratio $\delta\sigma/\delta\chi_d$ is expected to be of order $\mathcal{O}(1)$. Such a result changes in the presence of p-h

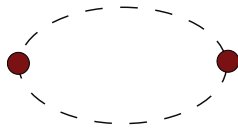


FIG. 2: (color online) AL contribution of the SCF to the current-current correlation function. The dashed lines represent the SCF propagator, while the red dots reduce to the constant c in the ordinary case Eq. (27), and to the momentum and frequency dependent vertex $c(q)$ in the presence of current-current interactions, see Eq. (32).

interactions. The elimination via gaussian integration of the current field ϕ^s from Eq. (24) leads to

$$\begin{aligned} S_{GL}(\mathbf{A}, \Delta) = & -\frac{1}{2} \chi_{jj}^{\alpha\beta} (1 + Z(q)) A_q^\alpha A_{-q}^\beta + \\ & + (g^{-1} - \Pi(\omega) + c\mathbf{q}^2) \Delta_q^* \Delta_q + \\ & - c(q) \mathbf{q} \mathbf{A}_{q'} (\Delta_q^* \Delta_{q-q'} + \Delta_{q+q'}^* \Delta_q) \end{aligned} \quad (28)$$

where

$$Z(q) = \frac{\chi_{jj}(q)/(2t\delta)^2}{1/g - \chi_{jj}(q)/(2t\delta)^2} \quad (29)$$

and

$$c(q) = c'(1 + Z(q)). \quad (30)$$

We notice that in Eq.s (28)-(30) we used a simplified notation valid in the $q \rightarrow 0$ limit relevant for the following discussion, so that the current-current correlation function χ_{jj} refers to its diagonal part $\chi_{jj}^{\alpha\alpha}$ only. At finite q the above expressions must be properly extended to account for the full matrix structure of the response functions. From the definition (28) and using Eq. (20) we compute the SCF contribution to the current-current response function. We find a correction to the mean-field value current-current response function (19) that leads to the RPA resummation of χ_{jj} as

$$\chi_{jj}^{RPA}(q) = \chi_{jj}(q) + Z(q)\chi_{jj}(q) = \frac{\chi_{jj}(q)}{1 - g\chi_{jj}(q)/(2t\delta)^2}, \quad (31)$$

and a second term that represents the SCF contribution, which generalizes the standard AL result (27) with a momentum and frequency dependent vertex $c(q)$:

$$\delta\chi^{\alpha\beta}(q) = T \sum_p c(q)^2 (2\mathbf{p} + \mathbf{q})_\alpha (2\mathbf{p} + \mathbf{q})_\beta L(p) L(p+q). \quad (32)$$

From Eq. (19) one can easily see that in the dynamic limit $\chi_{jj} = 0$, so that $Z = 0$ in Eq. (29) and from Eq. (30) $c(q) = c'$. In the opposite static limit instead we can rewrite χ_{jj} in Eq. (19) as

$$\begin{aligned} \chi_{jj} &= -\frac{2}{N} \sum_{\mathbf{k}} (\partial_{\mathbf{k}\alpha} \xi_{\mathbf{k}}^0)^2 \partial_{\xi} f(\beta \xi_{\mathbf{k}}) \\ &= \frac{2}{N} \sum_{\mathbf{k}} \frac{J_\alpha}{v_\alpha} (\partial_{\mathbf{k}\alpha}^2 \xi_{\mathbf{k}}^0) f(\beta \xi_{\mathbf{k}}) \end{aligned} \quad (33)$$

By direct comparison with the self-consistent equation (12) for the density field one immediately sees that $\chi_{jj} = 2t\delta(J_F/v_F)(\phi_0^c/g)$. From Eq. (29) it then follows that $1 + Z = v_F/J_F$, leading to $c(q) = c$ in Eq. (30). We then find that the vertices relative to the SCF contribution to the conductivity and diamagnetism are quantitatively different:

$$c(q) \propto v_F J_F \sim (2t\delta + \phi_0^c) 2\delta t \quad (\mathbf{q} = 0, \omega \rightarrow 0) \quad (34)$$

$$c(q) \propto v_F^2 \sim (2t\delta + \phi_0^c)^2 \quad (\mathbf{q} \rightarrow 0, \omega = 0). \quad (35)$$

The difference in the two limits reflects in a difference in the overall prefactors of the SCF contribution to the conductivity and diamagnetism. Indeed, from Eqs. (22)-(23) one has that

$$\frac{\delta\sigma}{|\delta\chi_d|} \propto \frac{((2t\delta + \phi_0^c) 2t\delta)^2}{(2t\delta + \phi_0^c)^4} \propto \delta^2 \quad (36)$$

i.e. the fluctuation conductivity is suppressed by the proximity to the Mott insulator by a factor that depends on the doping.

We notice that in the presence of HF corrections the gauge-invariant form of the GL functional for SCF is recovered in a non trivial way. Indeed, in the usual case the coupling to the gauge field \mathbf{A} can be obtained by the minimal-coupling substitution $\mathbf{q} \rightarrow \mathbf{q} - 2\mathbf{A}$ in the $\mathbf{q}^2\Delta^2$ term of the Gaussian propagator. This leads immediately to the term linear in \mathbf{A} , $c\mathbf{A} \cdot \mathbf{q}\Delta^2$, needed to compute the AL correction, see Eq.s (18)-(21). In our case by direct inspection of Eq. (24) one finds instead two different coefficients c, c' in the $\mathbf{q}^2\Delta^2$ and $\mathbf{A} \cdot \mathbf{q}\Delta^2$ terms, as we explained above. However, by integrating out the ϕ^s field the coupling of SCF to the gauge field is described in general by a term $c(q)\mathbf{A} \cdot \mathbf{q}\Delta^2$, (see Eq. (28)), where $c(q)$ is given by Eq. (30). As a consequence, one recovers the minimal-coupling prescription only in the static limit (35) where $c(q) = c^{21}$.

According to the discussion below Eq. (1) above, the result (36) can be recast as an estimate of the ratio between the correlation lengths extracted experimentally from paraconductivity and diamagnetism. For example using data reported in Ref. [13] one has that at $T \sim 24$ K, $\delta\sigma \sim 10^5(\Omega\text{m})^{-1}$, and $\delta\chi_d \sim 60$ A/mT. Using in Eq. (1) $\xi_0 \sim 1$ nm and $d \sim 15$ Å as appropriate for cuprates, we obtain that $k_B T/d\Phi_0^2 = 0.053$ A/mT and $e^2/16\hbar d\xi_0^2 = 10^4(\Omega\text{m})^{-1}$, so that $\delta\sigma/\delta\chi_d \propto \xi_\sigma^2/\xi_{\chi_d}^2 \sim 10^{-2}$. Such a strong suppression of the paraconductivity with respect to diamagnetism, that has been rephrased in Ref. [13] in terms of the vortex diffusion constant valid only in the case of KT fluctuations, can be more generally attributed from Eq. (36) to the overall δ^2 factor due to the proximity to the Mott-insulating phase. We checked that the estimate of the ratio (36) within the mean-field solution of the $t - J$ model is quantitatively larger by a factor ten than the one experimentally found, since a prefactor $\sim 2t/\phi_0^2$ partly compensate the δ^2 suppression. Such a quantitative disagreement is reminiscent of analogous limitations of the mean-field approach already discussed in the literature in the contexts of other physical quantities, as for example the scaling of the superfluid-density

depletion with doping¹⁹. Moreover, at low doping^{15,19,22} one should also include the boson fluctuations neglected so far, which give a temperature T_B for the boson condensation smaller than the one for the gap opening, leading to a suppression of the critical temperature T_c with respect to its mean-field value T_{MF} . In such a regime, the static limit of the $1 + Z$ correction in Eq. (30) above could not be simply given by v_F/J_F : nonetheless, the difference between the static and dynamic limit still holds, possibly leading only to a quantitative difference with respect to the result (36). Finally, we notice that the experimental estimate of the SCF contribution to paraconductivity reported in Ref. [13] is based on the scaling of the conductivity at frequencies large enough with respect to quasiparticle dissipation. As a consequence, it is worth making a comparison with the present results, that have been derived in the clean limit.

In summary, we analyzed the SCF contribution to conductivity and diamagnetism in the presence of current-current interactions, by using as a paradigmatic example the slave-boson formulation for the $t - J$ model. By explicitly constructing the GL fluctuation functional in the presence of HF corrections we showed that current-current interactions, needed to recover the gauge-invariant form of the GL functional, modify the transport coefficients leading to a momentum and frequency dependence of the vertex $c(q)$ entering the AL expression for the SCF contribution. Since different limits are involved in the definition of paraconductivity and diamagnetism, we obtain a different prefactor in the two cases, with a suppression of the paraconductivity because of the proximity to the Mott-insulating phase, as recently shown experimentally in cuprates¹³. Even though a mean-field approach to the $t - J$ model is not satisfactory from the quantitative point of view, the different strength of SCF contribution to conductivity and diamagnetism is more general, since it is a consequence of the existence of a sizeable difference between quasiparticle current and velocity. A quantitative comparison with experiments remains an interesting theoretical challenge, that certainly deserves further investigation.

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